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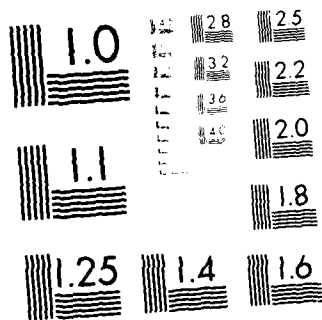
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A LOW-ORDER MODEL OF A MOIST GENERAL
CIRCULATION: FORMULATION AND TESTING

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31 January 1983

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highly truncated series of orthogonal functions. Quasi-geostrophic beta-plane dynamics are used. The diabatic processes are evaporation and precipitation, transfer of sensible and latent heat, and short-wave and long-wave radiation; solar radiation drives the circulation.

Preliminary numerical solutions show that the model produces qualitatively reasonable patterns, occurring in qualitatively reasonable sequence. Extended time intervals are sometimes required for initial transient conditions to die out. For suitably chosen solar heating, two stable equilibrium states - a cold state and a warm state - are alternative possible outcomes.

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1. Introduction

Many of the most important and influential recent works which dynamic meteorologists have accepted as belonging in their field, and which have been published in meteorological journals, have dealt with the behavior of a gas of uniform composition. By contrast, the most visible features of the real atmosphere, as is especially evident when one looks at satellite photographs of the earth, are the clouds, whose variable occurrence in space and time demands a fluid of varying composition. The clouds are not mere atmospheric ornaments; the fact that they are visible implies that they reflect much of the sun's radiation which would otherwise penetrate to lower levels in the atmosphere and would heat the ocean and land surfaces underneath. Moreover, the clouds, and the water vapor which must be present if the clouds are not to evaporate immediately, are strong absorbers and emitters of long-wave radiation, while the gain and release of latent heat, which must accompany changes in pressure if the air is to remain saturated, give moist air a different thermodynamical behavior from dry air. It is therefore relevant to pose the question, "To what extent is the dry atmosphere which appears in so many theoretical studies an acceptable model of an atmosphere where water in its gaseous, liquid, and solid phases occurs in varying concentrations?"

One can of course get some idea of the answer to this question by comparing the results of these theoretical studies with observations of the real atmosphere, or with other theoretical studies where the presence of water enters explicitly. The latter studies logically

include the recent work with large operational global circulation models. However, dynamicists often attack questions of this sort by constructing less complicated models, in which some real properties and processes deemed to have no bearing on these questions are purposely omitted. The most extreme models of this sort are the low-order models, formulated with a minimal number of dependent variables, where all but the essential features have been eliminated, and where care must often be taken not to eliminate some of the essentials as well.

Most low-order models of circulations of global scale have been dry models. It is easy to see why this has been so. In constructing a low-order model one usually strives for simplicity. Aside from the obvious fact that there are more basic dependent variables to handle in a moist model, both the radiative and the thermodynamic processes which characterize a moist atmosphere introduce complicated nonlinear terms into the governing dynamical equations. Low-order models are most frequently formulated in terms of spatially orthogonal functions, since, when sparsely distributed grid points are used, horizontal finite differences do not generally afford good approximations to horizontal derivatives. Simple products are easy to evaluate in orthogonal-function space, but more complicated nonlinear functions are not.

In large moist models which use orthogonal functions, this difficulty is generally circumvented by transforming the variables from orthogonal-function space to grid-point space at each time step, evaluating the cumbersome nonlinear terms at each grid point, and then

transforming back to orthogonal-function space, where the horizontal derivatives are evaluated. This procedure is also possible in a low-order moist model, but it introduces other difficulties. Because a global field of water vapor content possesses horizontal variations comparable to its mean value, the representation of a realistic field in terms of a small number of orthogonal function, with coefficients which are optimal for the bulk of the atmosphere, is likely to yield supersaturation when evaluated at some tropical grid points, and negative moisture content at some polar grid points. Neither outcome is tolerable.

In a Final Report entitled "The feasibility of a low-order model of a moist general circulation", prepared under a previous contract with the Air Force Geophysics Laboratory, we noted these difficulties, and described possible methods for overcoming them. In brief, we suggested eliminating the possibility of supersaturation by using a measure of total water content rather than water vapor content as a basic variable, and introducing an auxiliary equation to define the water vapor content in terms of the actual total water content and the water vapor content needed to produce saturation. We proposed to avoid the likelihood of negative moisture content by using the total dew point, i.e., the dew point which would result if all liquid water were converted to vapor, rather than the total-water mixing ratio as a basic variable; the variations of total dew point, like those of temperature, are generally small compared to mean values.

We then proceeded to describe in detail a low-order model of a moist general circulation incorporating the suggested features. We described some preliminary tests of the model for the special case of horizontally uniform solar heating. Tests with variable solar heating were yet to be performed.

During the two years since the appearance of that report we have made some substantial changes in the model. We have done so for a variety of reasons. In some instances the possibility of making the changes simply had not occurred to us. In other instances we had wished all along to make the changes, but had previously been unable to find a satisfactory way to do so. In still other instances we had not considered it desirable or necessary to make the changes, and were led to do so only after discovering that the model as originally formulated would not perform properly. Examples of changes made for these various reasons are, respectively, the use of vertical averaging to simplify the vertical structure of the model, the inclusion of the advection of temperature and moisture by the divergent part of the wind field, and the use of a grid of nine instead of sixteen points for evaluating the nonlinear radiative and thermodynamic terms.

Throughout the development of the model our goal has been simplicity. By this we mean simplicity in concept, simplicity in formulation, and simplicity in execution. It is not certain that we have achieved this goal; our model is much more complicated than a dry model with similar vertical and horizontal resolution.

Because of our attempts to simplify, we cannot expect the model to duplicate the behavior of the earth's atmosphere in more than a qualitative sense. We might better regard our model as a model of some hypothetical planet's atmosphere, possessing one constituent which occurs in two phases. However, when numerical values are required, we have chosen them to be representative of the earth's atmosphere, with water as the variable constituent.

Because we anticipate performing comparative numerical studies in which some constants or initial values are altered by very small amounts, we have made a point of eliminating all discontinuities in the formulation of the equations. In particular, we have used an unconventional formula for the water-vapor mixing ratio in terms of temperature and total water content.

In the following sections we shall present a detailed description of the construction of the model, including those features which we also described in the previous final report. We shall then examine some preliminary numerical solutions. Finally, we shall consider briefly some further modifications of the model which should perhaps eventually be made.

2. The continuous equations

Our model atmosphere will be composed of a mixture of dry air, water vapor, and liquid water in variable proportions. It will be of infinite upward and west-east extent, and will be bounded on the south

and north by frictionless vertical walls. Under the atmosphere will be an ocean of uniform composition and limited vertical extent. The atmosphere will be governed by baroclinic, geostrophic, beta-plane dynamics, while the ocean will exchange heat and water with the atmosphere. The sole external driving force will be incoming solar radiation; dissipation will be both thermal and mechanical. We shall not consider the possible presence of ice, even at subfreezing temperatures, nor shall we allow ocean currents to develop.

We shall begin by formulating the equations for a vertically and horizontally continuous atmosphere-ocean system. We shall then reduce the atmospheric equations to those of a two-layer model, and the oceanic equation to that of a single-layer ocean, by vertical averaging. Finally we shall introduce a truncated set of orthogonal functions, in terms of which each horizontally continuous field will be expressed.

Our basic equations will contain the constants

- D width of atmospheric channel, divided by π ,
- p_0 sea-level pressure,
- p_1 pressure at bottom of oceanic layer,
- f Coriolis parameter in middle of channel,
- β northward derivative of Coriolis parameter,
- c_p specific heat of air at constant pressure,
- R gas constant for air,
- L latent heat of condensation,
- c specific heat of water,

the independent variables

- t time,

x eastward distance,
y northward distance,
p pressure,

and the basic dependent variables

$\tilde{\psi}$ stream function,
 $\tilde{\chi}$ velocity potential (with sign changed),
 $\tilde{\omega}$ individual pressure change,
 \tilde{T} atmospheric temperature,
 \tilde{W} total dew point,
 \tilde{S} oceanic temperature.

The horizontal wind field will be the sum of a rotational nondivergent part, derivable from $\tilde{\psi}$, and an irrotational divergent part, derivable from $\tilde{\chi}$. To this list we shall add the auxiliary variables

\tilde{u} saturation mixing ratio at temperature \tilde{T} and pressure p ,
 \tilde{v} water vapor mixing ratio,
 \tilde{w} total-water mixing ratio,
 \tilde{s} saturation mixing ratio at temperature \tilde{S} and pressure p .

We have placed a tilde (\sim) over each dependent variable because we shall be using the same symbols, without tildes, for the values of these variables at the atmosphere-ocean interface.

The basic prognostic equations will be the vorticity equation

$$\partial \nabla^2 \tilde{\psi} / \partial t = -J(\tilde{\psi}, \nabla^2 \tilde{\psi}) - \rho \partial \tilde{\psi} / \partial x - f \nabla^2 \tilde{\chi} + F, \quad (1)$$

the atmospheric thermodynamic equation

$$d(c_p \tilde{T} + L \tilde{w})/dt = R \tilde{T} \tilde{\omega}/p + H, \quad (2)$$

the water-content equation

$$d\tilde{w}/dt = G, \quad (3)$$

and the oceanic thermodynamic equation

$$d(c \tilde{S})/dt = E. \quad (4)$$

Here J denotes a Jacobian with respect to x and y , while the diabatic terms F , H , G , and E denote respectively the curl of the viscous drag, the diabatic atmospheric heating per unit mass (including the effect of evaporation from the ocean surface, but not condensation within the atmosphere, which increases \tilde{T} but decreases \tilde{w}), the gain or loss of water by evaporation or precipitation, and the oceanic heating per unit mass. In keeping with the geostrophic simplification we have omitted the advection of vorticity by the divergent part of the wind, but the complete advection of enthalpy and water are implicitly included in the time derivatives in (2) and (3). These equations are to be accompanied by the diagnostic continuity and thermal-wind equations

$$\partial \tilde{\omega} / \partial p = -\nabla^2 \tilde{\chi}, \quad (5)$$

$$\partial \tilde{\Psi} / \partial p = -(R/f) \tilde{T} / p . \quad (6)$$

Eq. (6) implies that the horizontal average of $\tilde{\Psi}$ increases with elevation, and that, aside from a constant factor, $\tilde{\Psi}$ is the isobaric height. For boundary conditions we shall let $\tilde{\omega} = 0$ when $p = p_0$ or 0.

The auxiliary dependent variables are related to the basic variables by the formulas

$$\tilde{u} = c' \tilde{T}' / p , \quad (7)$$

$$\tilde{w} = c' \tilde{W}' / p , \quad (8)$$

$$\tilde{s} = c' \tilde{S}' / p , \quad (9)$$

$$(\tilde{u} - \tilde{v})(\tilde{w} - \tilde{v}) = \gamma^2 \tilde{v}^2 . \quad (10)$$

Here c' is to be chosen to produce the proper saturation mixing ratio at some standard temperature and pressure, and the choice of γ (and indeed the choice of Eq. (10) to define \tilde{v}) is somewhat arbitrary. Eqs. (7)-(9) may be derived from the Clausius-Clapeyron equation if the factor \tilde{T}^2 in that equation is first replaced by the product $T^* \tilde{T}$, where T^* is a standard temperature typical of the atmosphere, say 273 K. This approximation makes (8)-(10) algebraic instead of

transcendental. An appropriate value for the exponent μ is about 20.0, and by letting $\mu = 20$ exactly we can effect a considerable saving in computation. Eq. (10) makes the liquid water content $\tilde{w} - \tilde{r}$ small when the degree of subsaturation $\tilde{m} - \tilde{r}$ is large, and vice versa. Choosing $\gamma = 1/4$ makes the relative humidity \tilde{r}/\tilde{m} equal to 80 percent when \tilde{T} and \tilde{W} are equal; the remaining 20 per cent of the water is in the form of clouds. Choosing $\gamma = 0$ would reduce (10) to the conventional assumption that there is no supersaturation, and no clouds with subsaturation, but it would also introduce the usual discontinuity at the saturation point; we therefore prefer a positive γ . Eqs. (1)-(10) form a closed system in the ten basic and auxiliary dependent variables.

Our low-order model is to consist of a simpler system of equations, obtained by introducing simplifying assumptions into (1)-(10). However, before we alter the meaning of (1)-(10) at all, we can realize some simplification by expressing the constants and variables in dimensionless form. We can effectively do this by choosing $1/f$, D , p_0 , and $D^2 f^2 / R$ as the units in which time, distance, pressure, and temperature are measured. The numerical values of f , D , p_0 , and R are then unity, while the numerical values of the remaining constants are the ratios of these constants to the combinations of f , D , p_0 , and R with the appropriate dimensions. In the equations which follow we shall assume, unless we state otherwise, that all quantities have been made dimensionless, and we shall omit any factors whose numerical values must be unity.

3. Vertical simplification

In many two-level or two-layer geostrophic models the stream function is specified at each of two levels, or averaged through each of two layers, while the temperature is specified for only one level or layer. The temperature is then identified through the thermal-wind equation with the difference of the stream functions. We shall use the two-stream-function one-temperature format in our model. It is consistent with this formulation to specify the total dew point for only one level or layer.

Our procedure for reducing Eqs. (1)-(10) to a two-layer model will be dictated by a consideration of the source and sink terms F , H , and G . It is not at all obvious how these should be formulated at individual levels within the atmosphere. However, if horizontal dissipative exchanges of momentum, energy, and water in the atmosphere are considered negligible, the vertical averages of F , H , and G simply represent net exchanges between the atmosphere and the ocean, and, in the case of energy, between the atmosphere and outer space. Accordingly, we shall construct our vertically simplified model by averaging Eqs. (2)-(4) through the depth of the atmosphere or ocean. Eq. (1) must be averaged separately through each of two layers in order to yield two stream functions, whence the exchange of momentum between the two layers will also enter the model.

Because \tilde{u} , \tilde{v} , and \tilde{w} normally fall off much more rapidly with elevation than \tilde{T} and \tilde{W} , we cannot choose any single level and claim that the values of the various terms in (1)-(3) at this level are

representative of vertical averages. We shall therefore specify the manner in which each dependent variable varies with p , whereupon vertical integration will be feasible. We shall find it convenient to express the dependent variables in terms of their values at the atmosphere-ocean interface; these will be denoted by symbols without tildes, and will become our new dependent variables.

We first let the lapse rate of atmospheric temperature with elevation be constant, so that

$$\tilde{T} = T p^{\lambda}, \quad (11)$$

where λ is a constant. We make no attempt to model a stratosphere. Since $\lambda = 0$ would imply an isothermal stratification, and $\lambda = \chi = R/c_p$ would imply a dry-adiabatic lapse rate, λ should be between 0 and χ . We omit all vertical variations of oceanic temperature, since we are considering only a shallow layer, so that

$$\tilde{S} = S. \quad (12)$$

It follows from (6), (7), and (9) that

$$\tilde{\Psi} = \Psi + T(1 - p^{\lambda})/\lambda, \quad (13)$$

$$\tilde{\mu} = \mu p^{\lambda-1}, \quad (14)$$

$$\tilde{S} = S . \quad (15)$$

Next we let the relative humidity \tilde{v}/\tilde{u} be constant within each vertical column. Then

$$\tilde{v} = v p^{\mu\lambda-1} . \quad (16)$$

Eq. (10), being homogeneous in \tilde{u} , \tilde{v} , and \tilde{w} , still holds when the tildes are dropped, so that

$$\tilde{w} = w p^{\mu\lambda-1} , \quad (17)$$

whence, from (8),

$$\tilde{W} = W p^{\lambda} . \quad (18)$$

In order that \tilde{w} remain finite as the top the atmosphere is approached, it is necessary that $\lambda > \nu$, where $\nu = 1/\mu = 0.05$. Finally we let $\tilde{\chi}$, like $\tilde{\psi}$, be linear in p^{λ} , so that, if (5) is to be satisfied, with $\tilde{\omega} = 0$ when $p = 0$ or 1 ,

$$\tilde{\chi} = \chi (-1 + (1 + \lambda) p^{\lambda}) / \lambda , \quad (19)$$

$$\tilde{\omega} = \nabla^2 \chi (p - p^{1+\lambda}) / \lambda . \quad (20)$$

We may treat Eqs. (11)-(20) as approximations to be used only for vertical averaging, or we may assume that F , H , G , and E implicitly include internal processes which serve to maintain the prescribed vertical distributions against the disruptive effects of advection and other processes, just as vertical motion and divergence serve to maintain hydrostatic and geostrophic equilibrium against the disruptive effects of such processes.

We shall average Eq. (1) separately from $p = 0$ to 1 and $p = 0$ to a level λ' ; this is equivalent to averaging from 0 to λ' and λ' to 1 . The most logical value of λ' would probably be $1/2$, but it makes the equations slightly simpler to let $\lambda' = (1-\lambda)^{1/\lambda}$, or, for acceptable values of λ , about $1/3$. We then find that two linear combinations of the averaged equations are

$$\begin{aligned} \partial \nabla^2 \psi / \partial t = & -J(\psi, \nabla^2 \psi) + (1+2\lambda)^{-1} J(\tau, \nabla^2 \tau) - \beta \partial \psi / \partial x \\ & - \nabla^2 \chi + 2\bar{\bar{F}} - \bar{F} \end{aligned} \quad (21)$$

$$\begin{aligned} \partial \nabla^2 \tau / \partial t = & -J(\psi, \nabla^2 \tau) - J(\tau, \nabla^2 \psi) - (1+2\lambda)^{-1} (3+\lambda) J(\tau, \nabla^2 \tau) \\ & - \beta \partial \tau / \partial x + (1+\lambda) \nabla^2 \chi - (1+\lambda)(\bar{\bar{F}} - \bar{F}) \end{aligned} \quad (22)$$

where the double and single bars over F denote averages from 0 to 1 and 0 to λ' , respectively.

Averaging Eq. (2), after using (10), (7), and (3) to express $d\tilde{v}/dt$ in terms of $d\tilde{T}/dt$ and G , we obtain

$$\begin{aligned} & ((1+2\lambda) + \lambda(1+\nu)\alpha) \left(\frac{\partial T}{\partial t} + J(\psi, T) \right) = -(\lambda + (\lambda-\nu)\alpha) \nabla T \cdot \nabla \chi \\ & + ((\lambda-\nu) - \nu(\lambda-\nu)\alpha) T \nabla^2 \chi + (1+\lambda)(1+2\lambda) \left(\frac{\bar{H}}{c_p} - \frac{L}{c_p} \frac{\partial \nu}{\partial w} \bar{G} \right), \end{aligned} \quad (23)$$

where

$$\alpha = \frac{L}{c_p} \frac{(1+\lambda)(1+2\lambda)}{\lambda^2(1+\nu)} \frac{\partial \nu}{\partial w} \frac{w}{T}, \quad (24)$$

and the partial derivatives $\partial v / \partial u$ and $\partial v / \partial w$ are to be evaluated from (10). Averaging Eq. (3), and then using (8) to express w in terms of W , we obtain

$$\begin{aligned} & \lambda(1+\nu) \left(\frac{\partial W}{\partial t} + J(\psi, W) \right) + \nu J(T, W) = -(\lambda-\nu) \nabla W \cdot \nabla \chi \\ & - \nu(\lambda-\nu) W \nabla^2 \chi + \lambda^2(1+\nu) \frac{W}{w} \bar{G}. \end{aligned} \quad (25)$$

Finally, Eq. (4) becomes

$$dS/dt = \bar{E}/c. \quad (26)$$

With the auxiliary variables u , v , w , and s defined by (7)-(10), applied at $p = 1$, and with the definition (24), Eqs. (21)-(23) and (25)-(26) become a closed system in the basic dependent variables ψ , χ , T , W , and S .

4. Horizontal simplification

Low-order models of dry atmospheric circulations are typified by a two-layer model which we introduced some time ago (Lorenz, 1963) to study the phenomenon of vacillation, i.e., periodic behavior which cannot be reduced to steady behavior by introducing a moving coordinate system. In that model we expanded each dependent variable in a truncated series of orthogonal functions; specifically, in a somewhat different notation, we let

$$A = \sum_{n=0}^6 A_n \Phi_n, \quad (27)$$

where A stands for any dependent variable, and

$$\left. \begin{aligned} \Phi_0 &= 1 \\ \Phi_1 &= 2 \sin y \cos 2x \\ \Phi_2 &= 2 \sin y \sin 2x \\ \Phi_3 &= \sqrt{2} \cos y \\ \Phi_4 &= 2 \sin 2y \cos 2x \\ \Phi_5 &= 2 \sin 2y \sin 2x \\ \Phi_6 &= \sqrt{2} \cos 2y \end{aligned} \right\} \quad (28)$$

The orthogonal functions Φ_n were chosen so that

$$\nabla^2 \Phi_n = -a_n \Phi_n, \quad (29)$$

where $a_0 = 0$, $a_1 = a_2 = 5$, $a_3 = 1$, $a_4 = a_5 = 8$, and $a_6 = 4$. They satisfy the orthonormality conditions

$$\pi^{-2} \int_0^\pi \int_0^\pi \Phi_n \Phi_m dx dy = \delta_{mn} , \quad (30)$$

so that, if C is any quantity to be expanded in a truncated series,

$$C_n = \pi^{-2} \int_0^\pi \int_0^\pi C \bar{\Phi}_n dx dy . \quad (31)$$

We then substituted the expansion (27) for each dependent variable into the horizontally continuous equations. Upon expanding the left and right sides of the resulting equations in truncated series, using (31), and then equating the coefficients of $\bar{\Phi}_0, \dots, \bar{\Phi}_6$, we effectively converted each continuous equation into seven ordinary differential equations; these became the equations of the low-order model.

In the present model we shall express the dependent variables in terms of the same truncated set of orthogonal functions. We may, in fact, regard the present model as an extension of the earlier model to a moist atmosphere.

In the dry model the right-hand sides of the continuous equations contain nonlinear terms of the form $J(A,B)$, to be expanded into orthogonal functions. According to (31), if $C = -J(A,B)$,

$$\left. \begin{aligned} C_0 &= 0 \\ C_1 &= 5b[A,B]_{23} + 8b[A,B]_{56} \\ C_2 &= 5b[A,B]_{31} + 8b[A,B]_{64} \\ C_3 &= 5b[A,B]_{12} + 4b[A,B]_{45} \\ C_4 &= 8b[A,B]_{26} + 4b[A,B]_{53} \\ C_5 &= 8b[A,B]_{61} + 4b[A,B]_{34} \\ C_6 &= 8b[A,B]_{15} + 8b[A,B]_{42} \end{aligned} \right\} , \quad (32)$$

where $b = (16\sqrt{2}/15\pi)$, and $[A, B]_{mn}$ stands for $A_m B_n - A_n B_m$. The present model also contains terms of the form AB . From (31), if $C = AB$,

$$\left. \begin{aligned} C_0 &= \sum_{n=0}^6 A_n B_n \\ C_1 &= (A, B)_{01} - ((A, B)_{16} - (A, B)_{34})/\sqrt{2} \\ C_2 &= (A, B)_{02} - ((A, B)_{26} - (A, B)_{35})/\sqrt{2} \\ C_3 &= (A, B)_{03} + ((A, B)_{14} + (A, B)_{25} + (A, B)_{36})/\sqrt{2} \\ C_4 &= (A, B)_{04} - ((A, B)_{46} - (A, B)_{13})/\sqrt{2} \\ C_5 &= (A, B)_{05} - ((A, B)_{56} - (A, B)_{23})/\sqrt{2} \\ C_6 &= (A, B)_{06} - ((A, B)_{66} + A_1 B_1 + A_2 B_2 + A_3 B_3)/\sqrt{2} \end{aligned} \right\} (33)$$

where $(A, B)_{mn}$ stands for $A_m B_n + A_n B_m$. Formulas for C_n , where $C = \nabla A \cdot \nabla B$, can be derived from (29) and (33) by noting that

$$\nabla A \cdot \nabla B = \frac{1}{2} (\nabla^2 (AB) - A (\nabla^2 B) - (\nabla^2 A) B). \quad (34)$$

As already noted, to evaluate the more complicated nonlinear functions appearing in the source and sink terms, and in the moist-thermodynamic terms, it is preferable to transform from orthogonal functions to grid points, evaluate the nonlinear terms at each grid point, and then transform back. We have chosen a set of nine grid points, at the intersections of the lines $x = \pi/6, 3\pi/6, 5\pi/6$ with the lines $y = \pi/6, 3\pi/6, 5\pi/6$. Our choice of nine points has followed earlier experimentation with a grid of 16 points. We

discovered, for the case where there were no variations with x , that unreasonably high temperatures would develop if the number of grid points in the y -direction exceeded the number of orthogonal functions (here Φ_0, Φ_3, Φ_6) depending upon y alone. This difficulty disappeared when the number of grid points was reduced to the number of orthogonal functions. This result appears contrary to our experience with transformations between orthogonal functions and grid points when the nonlinear terms are quadratic, and apparently results from the higher-degree nonlinearity. Thus far we have encountered no difficulties from the use of nine rather than seven grid points, and, in any event, we have found no way to choose a reasonably spaced grid of seven points.

Our model contains no prognostic equation for χ . To integrate the equation, we must eliminate $\partial T / \partial t$ from (22) and (23), and solve the resulting diagnostic equation for χ . Although some of the terms must be evaluated in grid-point space, the equation, which contains horizontal derivatives of χ , is not suitable for solution in grid-point space, and special care must be taken to make it readily solvable even in orthogonal-function space.

We note first that if ∇^{-2} denotes the particular inverse of ∇^2 whose horizontal average vanishes, (22) and (23) may be written as

$$\partial(T - T_0) / \partial t = b'(\chi - \chi_0) + \nabla^{-2} F', \quad (35)$$

$$\chi' \partial T / \partial t = Y' \nabla T \cdot \nabla \chi + Z' T \nabla^2 \chi + H', \quad (36)$$

where F' and H' include all the terms in (22) and (23) which contain neither χ nor a time derivative. It is easy to eliminate $\partial T / \partial t$. Without loss of generality we can let $\chi_0 = 0$. According to (33) and (34), when the resulting equation is transformed to orthogonal-function space, terms of the form $A\chi$ or $A\nabla^2\chi$ will transform to linear expressions in the unknowns χ_1, \dots, χ_6 , whose coefficients are linear functions of A_0, \dots, A_6 . However, the term $Y'\nabla T \cdot \nabla \chi$ is not of the proper form. Accordingly, we first divide (36) by Y' , which fortunately never vanishes, and then subtract the proper multiple of (35), obtaining

$$\begin{aligned} & (X'/Y')(\partial T_0 / \partial t + b'\chi) - \nabla T \cdot \nabla \chi - (Z'T/Y')\nabla^2\chi \\ & = (H' - X'\nabla^2 F')/Y' . \end{aligned} \quad (37)$$

In orthogonal-function space this is a set of seven linear algebraic equations in $\partial T_0 / \partial t$ and χ_1, \dots, χ_6 , whose coefficients are obtained by transforming X'/Y' , T , and $Z'T/Y'$ to orthogonal-function space, and whose right-hand sides are obtained by transforming $(H' - X'\nabla^2 F')/Y'$. Once these algebraic equations have been solved, the integration of the model is straightforward.

5. Sources and sinks

As we have already noted, the vertical averages of F , H , and G represent vertical exchanges of momentum, sensible and latent heat, and water. We shall assume that the exchanges across the ocean-atmosphere interface are proportional to the differences of appropriate quantities across this interface, with the same factor of proportionality k . We could appeal to Ekman-layer theory to determine a suitable value for k , but, in view of the drastic simplifications already introduced, we can hardly justify anything more involved than simply choosing a time period, say five days, for $1/k$. Exchange of momentum across the surface $p = \lambda'$, where there is no Ekman layer, will be made proportional to the shear, with a damping coefficient k' considerably smaller than k .

To formulate precipitation we shall assume that clouds have a "half life", i.e., that during a certain time interval the clouds give up a certain fraction of their water as rain. The damping time $1/l$ should be considerably shorter than $1/k$, perhaps one day. Denoting the radiative contributions to H and E by H_r and E_r , we have

$$\bar{F} = -k \nabla^2 \psi, \quad (38)$$

$$\bar{F} = -k' \nabla^2 T, \quad (39)$$

$$\bar{H}/c_p = -k(T - S - (L/c_p)(w - S)) + \bar{H}_r/c_p, \quad (40)$$

$$\bar{G} = -k(\tau - s) - l(v/\lambda)(w - \tau) \quad , \quad (41)$$

$$\bar{E}/c = -c''k(S - T + (L/c_p)(s - \tau)) + \bar{E}_r/c \quad , \quad (42)$$

where $c'' = p_0 c_p / (p_1 - p_0)c$. The factor v/λ appears in the precipitation term in (41) because the vertical average of \tilde{w} is $(v/\lambda)w$.

Radiation is traditionally a complicated process, and, despite our efforts at simplification, our formulation will reflect this fact. Even if we are interested simply in an atmosphere with an absorbing constituent, the appropriate radiation formulas will be highly dependent upon what this constituent is, and our radiation formulas, to a greater extent than our other formulas, will be based upon the supposition that it is water.

We shall express the incoming solar radiation in terms of a planetary temperature T_q ; this is the temperature which a black body covering the entire sky would have to have, in order to be equivalent to the sun. We shall allow some of the incoming solar radiation to be reflected by clouds; the remainder will pass through the atmosphere and heat the ocean. The fractional cloud cover a , which to a first approximation will equal the reflected fraction of the solar radiation, will be a function of the relative humidity. This appears preferable to letting the cloud cover depend upon total water content; we are not aware, for example, that cloudiness is greater in the tropics than in the polar regions. A reasonably suitable function is $a = (v/u)^4$.

The ocean will in turn emit long-wave radiation as a black body. The cloud-free portion of the atmosphere will be assumed to possess a spectral window, through which a fixed fraction a' of the long-wave radiation will pass; the remaining long-wave radiation will be partially absorbed and reemitted by atmospheric water vapor. The cloudy portion of the atmosphere will behave like the cloud-free portion, except that there will be no spectral window.

Our expressions for emission of long-wave radiation will need to contain the atmosphere's fractional emissivity, and the temperature at which the emission occurs. In a model with such low vertical resolution, we balk at considering the radiative transfer from level to level, and find no reason for using anything more complicated than Simpson's method. Basically, Simpson (1928) treated the atmosphere as completely transparent in the 8.5-11 micron band (the window), while below 7 and above 14 microns he treated any layer containing 0.3 kg of water per m^2 of cross section as completely opaque. Simpson's atmosphere would therefore radiate to space in the latter wave lengths at the temperature of the uppermost such layer. Between 7 and 8.5, and between 11 and 14 microns, more water vapor would be needed to render the atmosphere opaque.

Paraphrasing Simpson's treatment, we shall as a first approximation allow the cloud-free portion of the atmosphere to radiate upward and downward, at temperatures T' and T'' respectively, with the fraction $1 - a'$ of the intensity of black-body radiation, while it will absorb the fraction $1 - a'$ of the radiation it receives from the ocean. We

shall neglect any variations of a' which ought to occur with temperature. We shall treat the cloud-covered portion of the atmosphere similarly, except that a' will be replaced by 0. The temperatures T' and T'' will occur at pressures p' and p'' , which will be the pressure levels above which and below which the amount of water vapor is $V^*/2$, where $V^* = 0.3 \text{ kg/m}^2$. Thus p' and p'' are supposed to represent the centers of the uppermost and lowermost layers whose water vapor content is V^* . Letting v^* be the value which v would possess if the water vapor content of an entire column were V^* , i.e., $v^* = \frac{1}{2} \lambda g V^* / p_0$, where g is the acceleration of gravity, we obtain, for our first approximation,

$$p' = \left(\frac{1}{2} n^* / n \right)^{v/\lambda}, \quad (43)$$

$$p'' = \left(1 - \frac{1}{2} n^* / n \right)^{v/\lambda}. \quad (44)$$

Eqs. (43) and (44) are fairly satisfactory if v/v^* is large, but if $v < v^*/2$, they place the uppermost layer with a water content of V^* below the lowermost layer, while in actuality such layers do not exist at all. We shall therefore modify (43) and (44) by replacing v by $v + v^*$. Introducing the quantity $v' = v/(v + v^*)$, which is near unity when v is large but is small when v is small, we find that

$$p' = \left(\frac{1}{2} - \frac{1}{2} n' \right)^{v'/\lambda}, \quad (45)$$

$$p'' = \left(\frac{1}{2} + \frac{1}{2} n' \right)^{v'/\lambda}, \quad (46)$$

whence

$$T' = T \left(\frac{1}{2} - \frac{1}{2} \nu' \right)^\nu, \quad (47)$$

$$T'' = T \left(\frac{1}{2} + \frac{1}{2} \nu' \right)^\nu. \quad (48)$$

Eqs. (45) and (46) are almost identical to (43) and (44) when ν is large, but they make p' and p'' approach one another as $\nu \rightarrow 0$.

The assumption that the intensity of the radiation is $1 - a'$ times that of black-body radiation also becomes unrealistic when ν is small, and there is very little water vapor to radiate. Likewise, a sufficiently tenuous cloud layer should not radiate as a black body. We shall adjust for this situation by multiplying the emitted and absorbed radiation by ν' . Finally, a tenuous cloud layer should not be a complete reflector of solar radiation, and we shall multiply a by ν' to obtain the cloud albedo.

Collecting our results, and letting $a'' = a + (1 - a)a'$, we find that

$$\bar{H}_r / c_p = (g \sigma c_p^{-1} p_o^{-1}) \nu' a'' (S^4 - T''^4 - T'^4), \quad (49)$$

$$\bar{E}_r / c = c'' (g \sigma c_p^{-1} p_o^{-1}) (-S^4 + \nu' a'' T''^4 + (1 - \nu' a) T_a^4), \quad (50)$$

where σ is the Stefan-Boltzman constant.

6. Preliminary numerical runs

We have developed a low-order model of a moist circulation for the purpose of performing some comparative numerical experiments, aimed at clarifying the importance of water as an atmospheric constituent. We anticipate comparing pairs of runs; the members of a single pair will be alike in most respects, but will differ in the value of one or possibly several parameters. Pairs whose members differ only in initial conditions, and which may therefore be expected to exhibit similar statistical properties, can be used as controls. Meanwhile, as is often the case when one introduces a new model, our first studies will be directed more toward telling us something about the model. Only when we feel that we understand the capabilities and limitations of the model can we reasonably expect to learn something about the atmosphere, or about atmosphere-like systems which the model attempts to simulate.

A necessary prelude to performing numerical runs is the selection of numerical values for the various constants. We have chosen our values with the earth's atmosphere in mind. However, to reduce the time needed for transient effects associated with the chosen initial conditions to disappear, we have made the oceanic layer interacting with the atmosphere very thin. We might even regard the underlying surface as wet land rather than ocean.

The dimensionless values of f , D , p_0 , and R are all unity. We could solve the equations without specifying dimensional values at all, but in that case it would be difficult to compare our results with the real atmosphere. We have therefore chosen dimensional values; they are

$1/f = 3$ hours, $D = 1830$ km, $p_0 = 1000$ mb, and $R = 287 \text{ m}^2\text{s}^{-2}\text{K}^{-1}$. These particular values have the advantage of making the unit for temperature exactly 100 degrees; thus no further conversions beyond a displacement of the decimal point are needed for atmospheric comparisons.

Dimensionless values of the remaining constants, or, where sufficient, combinations of constants, follow. We have used these values in our numerical runs except where we have indicated otherwise.

λ	0.175
ν	0.05
χ	2/7
L/c_p	25.0
β	0.3
c'	$0.0038/(2.73)^{20}$
γ	0.25
k	0.015
k'	0.001
$2\nu/\lambda$	0.03
c''	1.0
a'	0.5
N^*/λ	0.0006
$\sigma g(c_p p_0)$	0.000061

In our first runs we have made the heat received from the sun, as represented by T_q , independent of geographical location. Our interest in these runs is in the equilibrium state which will ultimately be approached. We anticipate that in this state T , W , and S will be horizontally uniform, and that there will be no motion, but the values of T , W , and S which should correspond to a particular value of T_q are not at all obvious. The problem is therefore non-trivial.

In the Final Report of our previous contract we investigated the same problem. We discovered to our surprise that, when T_q lay in the range 269 K-274 K, there were two stable equilibrium states, and also an intermediate unstable equilibrium. One of the stable equilibria was warm, and one was cold, relative to what might have been expected with these values of T_q . Outside of the range, high values of T_q produced only warm equilibria, and low values produced only cold equilibria. Since we have subsequently made a number of changes in the model, it is of interest to see whether multiple equilibria still occur.

In Fig. 1 the heavy curve shows the equilibrium values of T as a function of T_q . We see that there are still multiple equilibria. However, the range of values of T_q for which such equilibria are found has been narrowed to less than two degrees. One might say that the tendency for multiple equilibria to occur has almost disappeared.

The large dots on the curve indicate where numerical runs were made. The dot at $T_q = 275.3$, $T = 277.5$, representing an unstable equilibrium, was located by a successive approximation procedure; previously the existence of the unstable equilibria had merely been deduced on theoretical grounds.

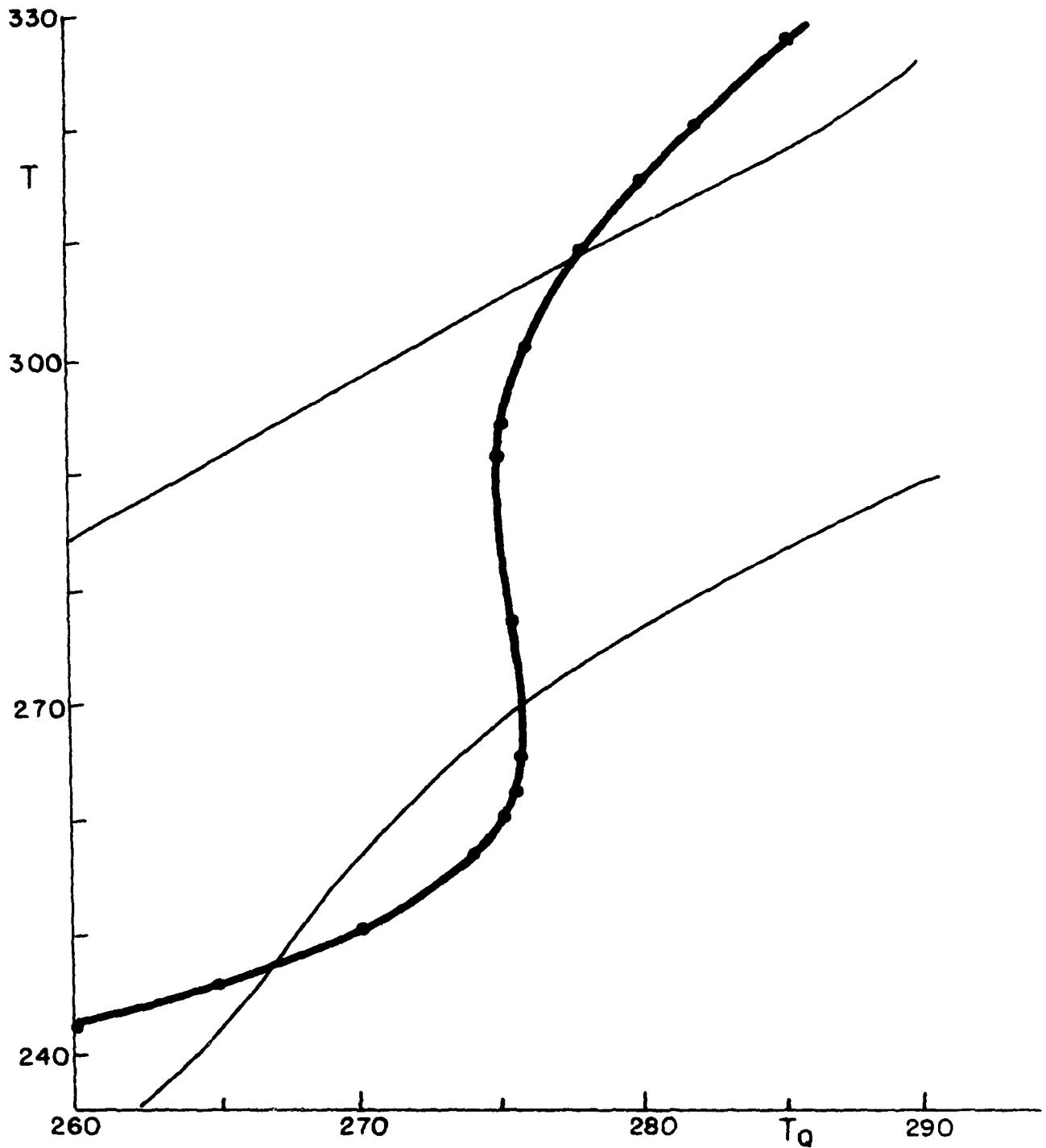


Fig. 1. Equilibrium values of temperature T corresponding to assigned values of planetary temperature T_0 , as given by the model. Heavy curve: albedo dependent on relative humidity; upper thin curve: fixed low albedo; upper heavy curve: fixed high albedo.

In the earlier report we hypothesized that the existence of multiple equilibria, and the accompanying steep slopes of the curve just outside the multiple-equilibrium range, resulted from a cloud-albedo feedback process; a paucity of clouds produces a low albedo, allowing more solar energy to be absorbed, thereby evaporating the clouds still more. We have attempted to place this hypothesis on firmer ground by performing some auxiliary runs where the dependence of the albedo upon the cloud amount has been eliminated. Instead, we have held the albedo fixed, in one set of runs at $0.4096 = 0.8^4$, and in a second set at $0.1296 = 0.6^4$. For determining the long-wave radiation, the dependence of the cloud amount upon the relative humidity has been retained.

The equilibrium states occurring under these conditions are indicated by the two thin curves in Fig. 1, the upper curve corresponding to the lower albedo. The tendency for multiple equilibria has completely disappeared, and, in fact, the upper curve is nearly a straight line, with T increasing by four degrees for every three-degree increase in T_g . The importance of the dependence of the albedo upon the cloud amount is thus confirmed.

Although our procedure for finding equilibria has been to choose arbitrary initial conditions and let the model run until equilibrium is approached, what we are really attempting to do when we seek an equilibrium state is to solve three simultaneous algebraic equations, two of which are highly nonlinear. These equations are obtained simply by equating H , G , and E to zero. The simplest of these equations, $G = 0$, expresses s as a weighted average of v and w ; with the

numerical values of k and Q which we have been using, $s = (v+w)/2$ when $G = 0$. As we noted in the earlier report, the most appropriate values for k and λ are uncertain in any case, and, by decreasing Q by a factor of 2, we reduce the equation $G = 0$ to the even simpler equation $s = w$, which implies that $S \approx W$. This result, when known, should facilitate the analysis and interpretation of accompanying results. Accordingly, we considered it desirable to examine the equilibrium curve which would occur when $\lambda v/\lambda = 0.015$ rather than 0.03.

The curve appears as the right-hand heavy curve in Fig. 2. The central heavy curve is the original equilibrium curve, copied from Fig. 1. The range of values of T_Q for which multiple equilibria occur has expanded enormously, and the difference between the two equilibrium values of T , when $T_Q = 290K$, is more than 80 degrees. One suspects that something about this curve is unrealistic, but, in any case, the multiple-equilibrium phenomenon is seen to be rather sensitive to the chosen numerical value of some of the constants.

The contrast between the curves suggests that multiple equilibria might disappear altogether if Q is increased rather than decreased. The left-hand heavy curve in Fig. 2 has been constructed from runs with $\lambda v/\lambda = 0.06$. There are indeed no multiple equilibria; there are only steep slopes.

The variations of the equilibrium curve are in qualitative agreement with what one might expect if a cloud-albedo process is operating. The low value of λ , which produces the widely separated equilibrium values, corresponds to clouds which precipitate their water

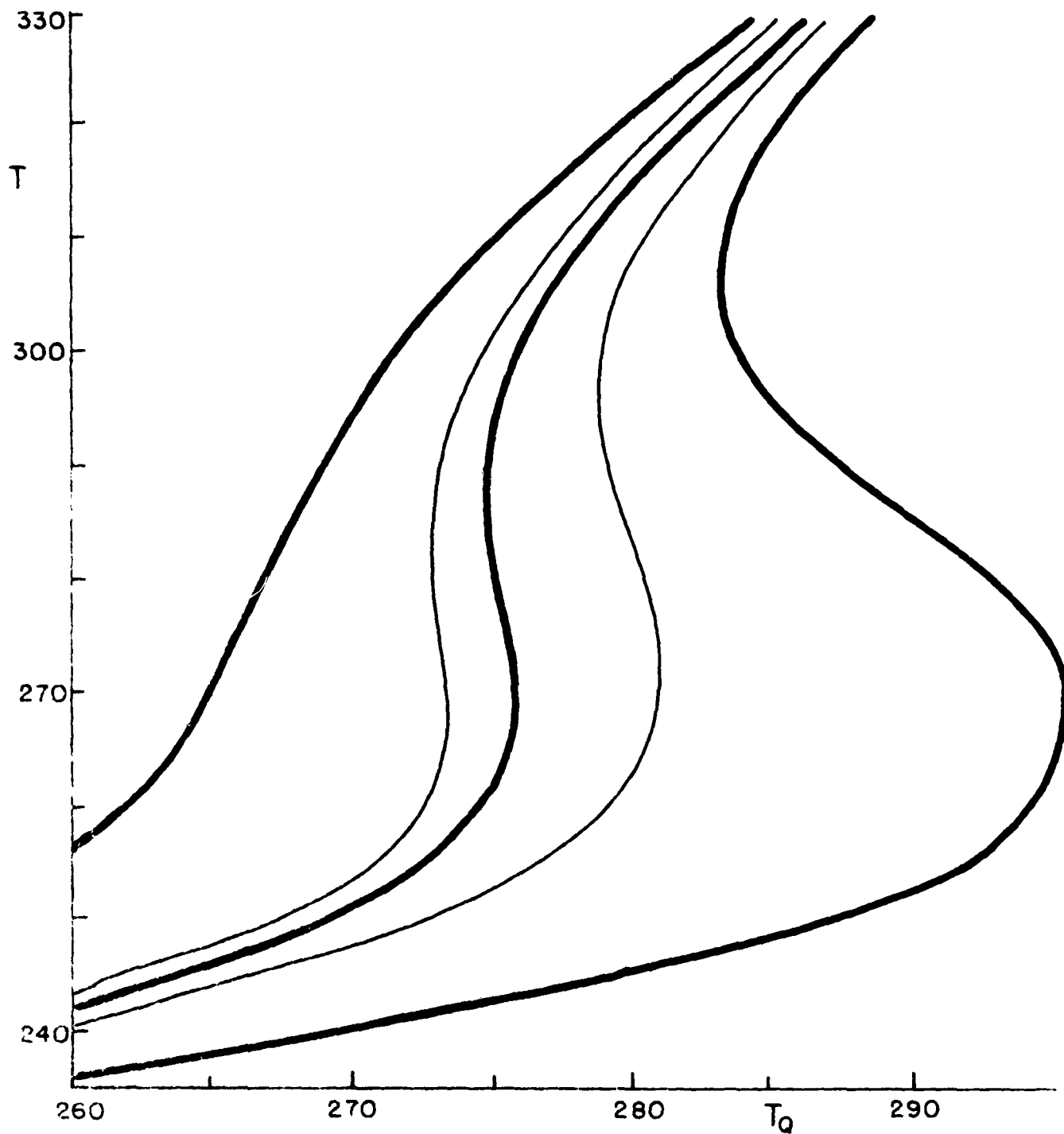


FIG. 2. Equilibrium values of temperature T corresponding to assigned values of planetary temperature T_0 , as given by the model, with albedo dependent on relative humidity, for various choices of parameters k and α affecting evaporation and precipitation. Left heavy curve: medium k , high α ; left thin curve: low k , medium α ; central curve: medium k , medium α ; right thin curve: high k , medium α ; right heavy curve: medium k , low α .

only with some difficulty, and which therefore tend to persist. The high value of λ corresponds to clouds which are rapidly attenuated by precipitation, so that the cloud-albedo feedback process cannot operate with sufficient strength to produce the multiple equilibria.

For good measure we have made two more sets of runs, corresponding to halved and doubled values of k , with the original value of λ . The results appear as the thin curves in Fig. 2. Qualitatively, decreasing k is like increasing λ , and vice versa, but the effect is not nearly so great. Something more than the simple ratio λ/k is involved; the heavy and the thin right-hand curves, which are far from identical, both result from runs with $\lambda\nu/\lambda = k$, and so with $S = W$.

Our final run allows for horizontal variations and accompanying motions; it has been made with $T_Q = 277.0 + 4.0\Phi_3$ (in °K). The initial conditions were obtained by superposing a fairly small perturbation $\Delta\psi = 0, \Delta T = \Delta W = \Delta S = \Phi_2$ on the zonal (but not steady) state $\psi = 0, T = W = S = T_Q$. Table 1 shows the development of the temperature field, expressed in terms of orthogonal functions.

Because the mean value of T_Q lies close to the multiple-equilibrium range, the low and high latitudes, where T_Q exceeds or falls short of 277.0 by several degrees, correspond to very high and very low equilibrium temperatures. Consequently T_3 , which represents the south-north temperature contrast and the accompanying westerly wind shear, increases from its initial value. During the first month the waves, represented by T_1, T_2, T_4 , and T_5 , damp considerably and, as indicated by the succession of positive and negative values, progress slowly from west to east.

Table 1. Values of T at selected times in a particular numerical integration of the low-order model, with $T_0 = 277.0 + 4.0 \Phi_3$

$t(\text{days})$	T_0	T_1	T_2	T_3	T_4	T_5	$T_6(^{\circ}\text{K})$
0	277.00	1.00	0.00	4.00	0.00	0.00	0.00
4	276.32	0.57	0.56	4.49	0.00	0.01	0.01
8	276.28	-0.10	0.65	4.73	-0.01	0.00	0.01
12	276.32	-0.46	0.28	4.93	0.00	-0.02	0.02
16	276.38	-0.35	-0.16	5.16	0.02	0.00	0.03
20	276.44	0.00	-0.28	5.38	0.00	0.02	0.03
24	276.51	0.20	-0.08	5.58	-0.02	0.00	0.03
28	276.57	0.11	0.11	5.80	0.00	-0.01	0.03
32	276.64	-0.05	0.09	6.01	0.00	0.00	0.03
36	276.70	-0.07	-0.03	6.22	0.00	0.00	0.03
48	276.90	-0.01	0.03	6.86	0.00	0.00	0.02
60	277.09	0.01	0.01	7.50	0.00	0.00	0.01
72	277.28	0.02	0.00	8.15	0.00	0.00	0.00
84	277.46	0.03	0.04	8.80	0.00	0.00	-0.01
96	277.60	-0.25	0.04	9.43	0.02	0.00	0.00
108	277.63	1.70	-0.78	9.15	0.04	-0.14	0.23
120	272.61	1.40	1.04	7.34	0.44	0.18	-0.96
132	270.98	1.09	0.22	8.02	0.06	0.00	-0.49
144	270.07	0.53	0.88	8.34	0.04	0.05	-0.40
156	269.03	-0.68	1.02	8.41	-0.02	0.15	-0.34
168	267.66	-1.33	-0.55	8.37	-0.16	-0.05	-0.34
180	266.34	0.01	-1.28	8.36	-0.05	-0.12	-0.34
240	260.86	1.07	-1.03	8.43	0.07	-0.06	-0.20
300	257.44	1.23	0.59	8.38	-0.08	-0.02	0.27
360	257.15	0.53	0.71	8.25	-0.14	-0.11	1.22
420	259.02	-0.32	0.36	8.44	0.04	-0.11	1.65
480	260.77	0.37	0.31	8.92	-0.13	-0.04	1.94
600	263.49	0.22	-0.40	10.21	-0.04	0.17	2.44
720	265.43	0.27	0.42	11.34	-0.16	-0.14	2.70
840	266.43	-0.06	-0.34	11.59	-0.26	0.43	2.65
960	265.00	0.13	-0.69	10.10	0.11	0.02	2.26
1080	265.00	0.51	0.16	10.10	-0.07	-0.23	2.26

During the third month, at about the time when T_3 reaches 8.5 degrees, the flow becomes baroclinically unstable, and the waves are suddenly rejuvenated. The waves derive their energy from the zonal energy (kinetic or available potential), so that by the end of the fourth month T_3 has fallen below its critical value, and the waves have ceased growing. For the remainder of the first year T_3 , and the amplitude of the waves, remain relatively constant.

A feature of the earlier runs which is apparent also in this run is the long time which is sometimes needed for transient effects to die out. With the rejuvenation of the waves, the mean temperature T_0 suddenly drops, and continues, until by the end of one year it has fallen a full 20 degrees. Presumably the waves have something to do with the decrease of T_0 , but the effect is not what one would expect in a dry model.

At about this time another phenomenon appears; T_6 , which has been rather small, becomes appreciably positive. A positive value of T_6 , together with a positive value of T_3 , means that the latitude of maximum westerly shear has shifted from the center of the channel toward the south. Evidently the current is less unstable baroclinically in its new position; before the end of the second year T_3 has increased by three degrees, with no accompanying intensification of the waves. The mean temperature has also recovered some of its loss, which supports the suggestion that the waves may have played a role in the loss.

Few variations are evident in the latter half of the third year other than the progression of the waves; in fact, the day-to-day values,

not presented in Table 2, show almost no variations. Apparently the transient effects have finally disappeared.

It is surprising, and disturbing from the point of view of computational economy, that years are required for the types of change that one might have expected to occur in months. However, aside from the slow decay times, the model appears to behave reasonably; waves develop when the zonal flow is sufficiently strong, and they progress in a reasonable manner. We believe that the model is ready for the initial "production runs".

7. Concluding remarks

As we have noted, our intention is to use our model in its present form to investigate the importance of water as an atmospheric constituent. Meanwhile, it is not inappropriate to anticipate still further changes. Even though we may feel that we have devoted enough time to the development process, there remains the possibility that the model, when put to use, will prove to be lacking in some essential element.

Perhaps the horizontal resolution will prove to be too low. Fortunately we have constructed the model so that we can increase the number of orthogonal functions and the accompanying number of grid points without making any other changes. Perhaps we shall have to formulate the clouds in a different manner.

We believe, however, that the most serious shortcoming, from a theoretical point of view, is the assumption of a fixed lapse rate of temperature with elevation. Amplifying waves should limit their ultimate growth by making the flow on which they are superposed less unstable baroclinically. One way in which they can do this is by reducing the north-south temperature contrast; our model allows them to do this. Another way is by transporting heat from low to high elevations, thereby stabilizing the lapse rate. A model admitting this possibility would be more realistic.

One possible scheme, which is the one which we used in our vacillation study with a dry model, is to allow the lapse rate to vary with time but not spatially. To make our present model capable of doing this, i.e., to make λ a function of time alone instead of a constant, we would need to make some assumption concerning the transfer of energy from one level in the atmosphere to another by radiation and other processes - an assumption which we have been expressly trying to avoid. Also, in our present model we have stored a number of functions of λ as constants; these would become variables. Nevertheless, we see no insurmountable difficulties.

As already stated, we believe that our immediate task is to make some production runs. Eventually, however, we believe that we should repeat some of these runs with a model with a variable λ , to see whether the conclusions continue to stand.

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